Model of Large Mixing Angle MSW Solution

Morimitsu TANIMOTO
Science Education Laboratory, Ehime University,
Matsuyama, Japan, 790-8577

February 1, 2008

Abstract

We have obtained the neutrino mass matrix with the large mixing angle (LMA) MSW solution, $\sin^2 2\theta_\odot = 0.65 \sim 0.97$ and $\Delta m^2_\odot = 10^{-5} \sim 10^{-4}\text{eV}^2$, in the $S_{3L} \times S_{3R}$ flavor symmetry. The structure of our neutrino mass matrix is found to be stable against radiative corrections.

The solar neutrino data as well as the atmospheric one give big impact on the study of the lepton mass matrices. There is a typical texture of the lepton mass matrix with the nearly maximal mixing of flavors, which is derived from the symmetry of the lepton flavor democracy [1, 2], or from the $S_{3L} \times S_{3R}$ symmetry of the left-handed Majorana neutrino mass matrix [3, 4]. This texture have given a prediction for the neutrino mixing $\sin^2 2\theta_{\text{atm}} = 8/9$. The mixing for the solar neutrino depends on the symmetry breaking pattern of the flavor such as $\sin^2 2\theta_\odot = 1$ or $\ll 1$. However, the LMA-MSW solution, $\sin^2 2\theta_\odot = 0.65 \sim 0.97$ and $\Delta m^2_\odot = 10^{-5} \sim 10^{-4}\text{eV}^2$ [5], has not been obtained in the previous works [1, 2, 3, 4]. We study how to get the LMA-MSW solution in the $S_{3L} \times S_{3R}$ symmetric mass matrices, and discuss the stability of the neutrino mass matrix against radiative corrections.

The texture of the charged lepton mass matrix was presented based on the $S_{3L} \times S_{3R}$ symmetry as follows [1, 3, 6]:

$$M_\ell = \frac{c_\ell}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + M_\ell^{(c)} \ , \tag{1}$$

where the second matrix is the flavor symmetry breaking one. The unitary matrix V_ℓ, which diagonalizes the mass matrix M_ℓ, is given as $V_\ell = FL$, where

$$F = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -2/\sqrt{6} & 1/\sqrt{3} \end{pmatrix} \tag{2}$$

diagonalizes the democratic matrix and L depends on the mass correction term $M_\ell^{(c)}$.

The neutrino mass matrix is different from the democratic one if they are Majorana particles. The S_{3L} symmetric mass term is given as follows:

$$c_\nu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + c_\nu r \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} , \tag{3}$$

where c_ν and r are arbitrary parameters. The eigenvalues of this matrix are easily obtained by using the orthogonal matrix F in eq.(2) as $c_\nu(1,\ 1,\ 1+3r)$.

The simplest breaking terms of the S_{3L} symmetry are added in (3,3) and (2,2) entries. Therefore, the neutrino mass matrix is written as

$$M_\nu = c_\nu \begin{pmatrix} 1+r & r & r \\ r & 1+r+\epsilon & r \\ r & r & 1+r+\delta \end{pmatrix} , \tag{4}$$

in terms of small breaking parameters ϵ and δ. In order to explain both solar and atmospheric neutrinos in this mass matrix, $r \ll 1$ should be satisfied. However, there is no reason why r is very small in this framework. In order to answer this question, we need a higher symmetry of flavors such as the $O_{3L} \times O_{3R}$ model [7].

Let us consider the case of $\delta \gg \epsilon \simeq r$, in which S_{3L} symmetry is completely broken. Then neutrino mass eigenvalues are given as

$$m_1 \simeq 1 + \frac{1}{2}\epsilon + r - \frac{1}{2}\sqrt{\epsilon^2 + 4r^2} , \quad m_2 \simeq 1 + \frac{1}{2}\epsilon + r + \frac{1}{2}\sqrt{\epsilon^2 + 4r^2} , \quad m_3 \simeq 1 + r + \delta, \tag{5}$$

in the c_ν unit. The orthogonal matrix U_ν is given as

$$U_\nu \simeq \begin{pmatrix} t & \sqrt{1-t^2} & \frac{r}{\delta} \\ -\sqrt{1-t^2} & t & \frac{r}{\delta-\epsilon} \\ \frac{r}{\delta}(\sqrt{1-t^2} - t) & -\frac{r}{\delta-\epsilon}(t + \sqrt{1-t^2}) & 1 \end{pmatrix} , \quad t^2 = \frac{1}{2} + \frac{1}{2}\frac{\epsilon}{\sqrt{\epsilon^2+4r^2}} . \tag{6}$$

Since the correction term L is close to the unit matrix, the MNS matrix $U_{\alpha i}$ is approximately given as $F^T U_\nu$ as follows:

$$F^T U_\nu \simeq \begin{pmatrix} \frac{1}{\sqrt{2}}(t + \sqrt{1-t^2}) & \frac{1}{\sqrt{2}}(\sqrt{1-t^2} - t) & -\frac{1}{\sqrt{2}}\frac{\epsilon r}{\delta(\delta-\epsilon)} \\ \frac{1}{\sqrt{6}}(t - \sqrt{1-t^2})(1 + \frac{2r}{\delta}) & \frac{1}{\sqrt{6}}(t + \sqrt{1-t^2})(1 + \frac{2r}{\delta-\epsilon}) & -\frac{2}{\sqrt{6}}(1 - \frac{r}{\delta}) \\ \frac{1}{\sqrt{3}}(t - \sqrt{1-t^2})(1 - \frac{r}{\delta}) & \frac{1}{\sqrt{3}}(t + \sqrt{1-t^2})(1 - \frac{r}{\delta-\epsilon}) & \frac{1}{\sqrt{3}}(1 + \frac{2r}{\delta}) \end{pmatrix} . \tag{7}$$

The mixing angle between the first and second flavor depends on t, which is determined by r/ϵ. It becomes the maximal angle in the case of $t = 1$ ($r/\epsilon = 0$) and the minimal one in the case of $t = 1/\sqrt{2}$ ($\epsilon/r = 0$). Since we get $\sin^2 2\theta_\odot = \epsilon^2/(\epsilon^2 + 4r^2)$, the relevant value of r/ϵ leads easily to $\sin^2 2\theta_\odot = 0.65 \sim 0.97$, which corresponds to the LMA-MSW solution. The numerical results have been shown in ref. [8].

We should carefully discuss the stability of our results against radiative corrections since the model predicts nearly degenerate neutrinos. When the texture of the mass matrix is given at the $S_{3L} \times S_{3R}$ symmetry energy scale, radiative corrections are not negligible at the electoroweak (EW) scale.

Let us consider the basis, in which the mass matrix of the charged leptons is diagonal. The neutrino mass matrix in eq.(4) is transformed into $V_\ell^\dagger M_\nu V_\ell$. The radiatively corrected mass matrix in the MSSM at the EW scale is given as $R_G \overline{M}_\nu R_G$, where R_G is given by RGE's [9] as

$$R_G \simeq \begin{pmatrix} 1+\eta_e & 0 & 0 \\ 0 & 1+\eta_\mu & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \eta_i = 1-\sqrt{\frac{I_i}{I_\tau}}\,(i=e,\ \mu), \quad I_i \equiv \exp\left(\frac{1}{8\pi^2}\int_{\ln(Mz)}^{\ln(M_R)} y_i^2\, dt\right). \tag{8}$$

We transform back this neutrino mass matrix $R_G \overline{M}_\nu R_G$ into the basis where the charged lepton mass matrix is the democratic one at the EW scale as follows:

$$F R_G \overline{M}_\nu R_G F^T \simeq c_\nu \begin{pmatrix} 1+\overline{r} & \overline{r} & \overline{r} \\ \overline{r} & 1+\epsilon+\overline{r} & \overline{r} \\ \overline{r} & \overline{r} & 1+\delta+\overline{r} \end{pmatrix} + 2\eta_R c_\nu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \overline{r} = r - \frac{2}{3}\eta_R\ . \tag{9}$$

Here we take $\eta_R \equiv \eta_e \simeq \eta_\mu$, which is a good approximation [9]. Its numerical value depends on $\tan\beta$ as: 10^{-2}, 10^{-3} and 10^{-4} for $\tan\beta = 60,\ 10$, and 1, respectively. As seen in eq.(4) and eq.(9), radiative corrections are absorbed into the original parameters r, ϵ and δ in the leading order. Thus the structure of the mass matrix is stable against radiative corrections although our model leads to nearly degenerate neutrinos. Of course, it does not means that the radiative corrections are small.

We have obtained the neutrino mass matrix with the large mixing angle MSW solution, $\sin^2 2\theta_\odot = 0.65 \sim 0.97$ and $\Delta m^2_\odot = 10^{-5} \sim 10^{-4}\mathrm{eV}^2$, in the $S_{3L} \times S_{3R}$ flavor symmetry. The structure of our neutrino mass matrix is found to be stable against radiative corrections. We wait for results in KamLAND experiment as well as new solar neutrino data.

References

[1] H. Fritzsch and Z. Xing, Phys. Lett. **B372**, 265(1996); ibid. **B440**, 313(1998).

[2] G. C. Branco, M. N. Rebelo and J. I. Silva-Marcos, Phys. Lett. **428B**, 136(1998).

[3] M. Fukugita, M. Tanimoto and T. Yanagida, Phys. Rev. **D57**, 4429(1998);
M. Tanimoto, Phys. Rev. **D59**, 017304-1(1999).

[4] K. Kang, S. K. Kang, J. E. Kim and P. Ko, Mod. Phys. Lett. **A12**, 1175(1997).

[5] J. N. Bahcall, P. I. Krastev and A. Yu. Smirnov, Phys. Rev. **D58**, 096016(1998).

[6] Y. Koide, Phys. Rev. D **28**, 252(1983); **39**, 1391(1989).

[7] M. Tanimoto, T. Watari and T. Yanagida, Phys. Lett. **461B**, 345(1999).

[8] M. Tanimoto, hep-ph/0001306.

[9] N. Haba, Y. Matsui, N. Okamura and M. Sugiura, Eur. Phys. J. **C10**, 677(1999).